



## Practical round. Problems to solve

### 7. Asteroid.

Analysis of observations of a near earth asteroid.

Astronomers of two observatories, which are located at a distance of 3172 km from each other, took CCD images of a certain region of the sky for the search of a near earth asteroid. Two images were obtained by Observatory 1 during the same night at 4<sup>h</sup>53<sup>m</sup> UT and at 7<sup>h</sup>16<sup>m</sup> UT. These images (negatives) are shown in Figs. 7.1 and 7.2, respectively. The next two images obtained on the same night were made at Observatory 1 and Observatory 2 simultaneously. These images (negatives) are shown in Figs. 7.3 and 7.4. The scale of all the images is the same as shown in Fig. 7.1.

7.1. Identify and mark the asteroid in the given Figs.

7.2. Measure the angular displacement (in arcsec) of the asteroid as seen from Observatory 1 and calculate its angular velocity in arcsec/s.

7.3. Measure the parallax of the asteroid (in arcsec) and calculate its distance from the earth.

7.4. Calculate the tangential linear velocity (velocity perpendicular to the line of sight) of the asteroid.

Note: You are provided a transparency for measurements of angular displacements of the asteroid.



код	
code	

7. Asteroid.

7. Астероид.

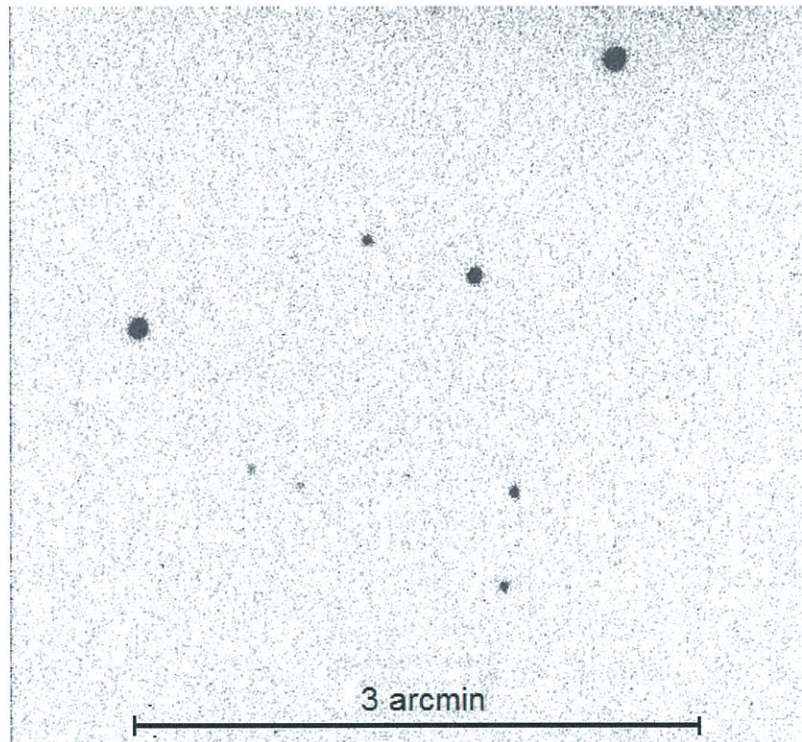


Fig. 7.1. 4:53 UT Рис. 7.1.

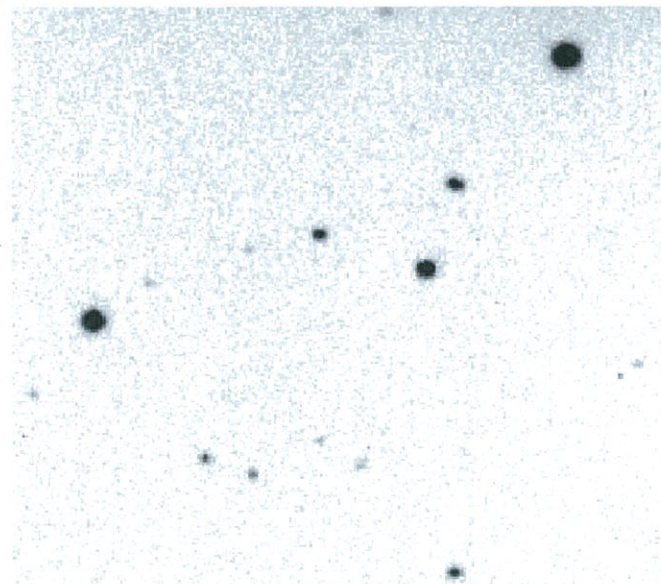


Fig. 7.2. 7:16 UT Рис. 7.2.



код	
code	

7. Asteroid.

7. Астероид.

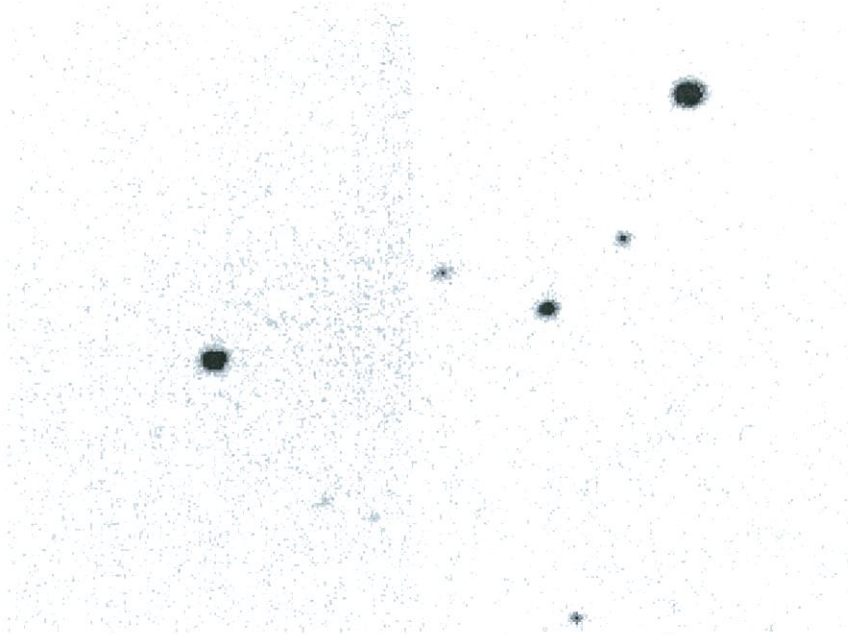


Fig. 7.3. Observatory 2 Рис. 7.3.

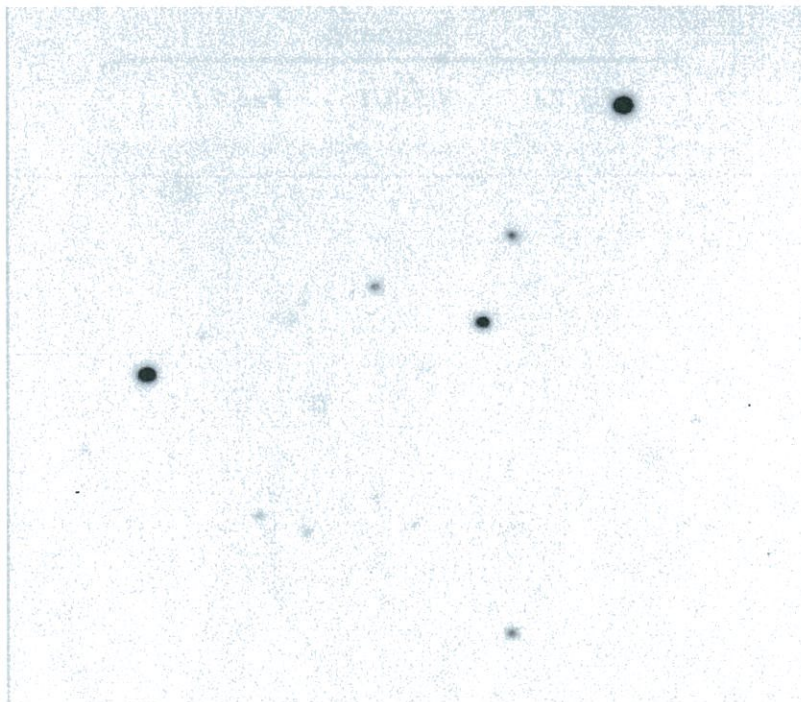


Fig. 7.4. Observatory 1 Рис. 7.4.

### Solution

- 1) Comparison of Figs. 7.1 and 7.2 allow us to identify the asteroid and evaluate its angular displacement due to its motion. We put the transparency on Fig.7.1 and mark positions of several bright objects with a pencil. Then we put the transparency on Fig. 7.2 and move it until the markings of objects on the transparency will coincide with positions of corresponding objects in Fig. 7.2. We notice that one object changed its position considerably in Fig. 7.2. This is the asteroid.
- 2) We mark the position of the asteroid on the transparency. Then we measure the displacement of the asteroid on the transparency and get  $\Delta l = 44$  mm. Using the scale of Fig. 7.1 we calculate the angular displacement of the asteroid.

$$\Delta\theta = 180 \times 44 / 81 = 98 \text{ arcsec}$$

The time interval between images of Figs.7.1 and 7.2 is  $\Delta t = 7\text{h}16\text{m} - 4\text{h}53\text{m} = 2\text{h}23\text{m} = 8580$  s.

The angular velocity of the asteroid  $\mu = \Delta\theta / \Delta t = 98 / 8580 = 0.011$  arcsec/s.

- 3) The parallax of the asteroid is evaluated by comparison of Figs. 7.3 and 7.4. These two images were taken from two different locations at the same time. We mark the positions of the asteroid overlaying the same transparency on Figs. 7.3 and 7.4. and measure the displacement of the asteroid on the same transparency,  $\Delta b = 7$  mm.

Angular displacement of the asteroid is  $\Delta\phi = 180 \times 7 / 81 = 16$  arcsec. Then the parallax  $p = \Delta\phi / 2 = 8$  arcsec.

The distance of the asteroid is  $d = B / (2 \tan p)$ , where B is the baseline, i.e. the distance between observatories. Then  $d = 206265 \times B / (2 p)$  since  $p$  is a small angle and measured in arcsec.

$$d = 206265 \times 3172 / (2 \times 8) = 40900000 \text{ km} = 0.27 \text{ au.}$$

- 4) The tangential linear velocity of the asteroid is  $v = d \tan \mu$  or  $v = d \mu / 206265$ .

$$v = 40900000 \times 0.011 / 206265 = 2 \text{ km/s.}$$



XVIII Международная астрономическая олимпиада

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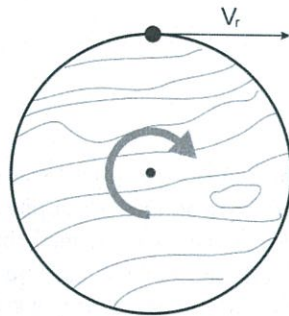
Литва, Вильнюс

6 – 14. IX. 2013

Vilnius, Lithuania

8.

A.



$$V_r = 12.6 \text{ km/s}$$

## Solution

Table A

<b>N [nm/mm] =</b>	<b>(659.39-654.62)/215.5 ≈ 0.0221</b>		
<b>λ [nm]</b>	<b>dx [mm]</b>	<b>dλ[nm]</b>	<b>v<sub>r</sub> [km/s]</b>
654.62	~5	~0.1105	~12.65
656.92	~5	~0.1105	~12.61
659.26	~5	~0.1105	~12.56
<b>v<sub>r_avg</sub> [km/s] =</b>			<b>~12.6</b>

### FORMULAE

Because of the Doppler effect the wavelength shift ,  $d\lambda$ , observed at the edge of Jupiter's disk can be obtained through the following formula:

$$v_r = \frac{1}{4} \cdot \frac{d\lambda}{\lambda} \cdot c$$

,where  $c$  – speed of light in the vacuum.

Coefficient  $\frac{1}{4}$  comes into effect due to the fact that:

- the sunlight, illuminating the planet, is reflected, therefore it was affected by Doppler effect twice,
- measurements are done at the edges of the disk and not relative to the non-shifted center, thus doubling the measured shift value.

Table B

dt[s] = 6695

Feature	x <sub>1</sub> [mm]	x <sub>2</sub> [mm]	L <sub>x</sub> [mm]	φ [°]
1	66	33	92	66.86
2*	57	46	92.5	67.86
3*	44	54	87	68.75
φ_avg =				~ 67.8

*\*only as example - actual values depends on choosen feature and accuracy of measuremets*

P<sub>Je</sub> [h] = ~ 9.9

R<sub>Je</sub> [km] = ~71470

FORMULAE

$$\varphi = \arcsin\left(\frac{x_1}{L_x}\right) + \arcsin\left(\frac{x_2}{L_x}\right)$$

$$P = \frac{360^\circ}{3600 \text{ s}} \times \frac{dt}{\varphi}$$

$$R_{Je} = \frac{v_r \cdot P_{Je}}{2\pi}$$

**Table C**

Moon	$P_m$ [h]	$a_{Je}$	$a$ [m]	$M_J$ [kg]
1	~ 43	~ 5.9	~ $4.22 \times 10^8$	~ $1.86 \times 10^{27}$
2	~ 85	~ 9.4	~ $6.70 \times 10^8$	~ $1.90 \times 10^{27}$
3	~ 172	~ 14.9	~ $10.65 \times 10^8$	~ $1.86 \times 10^{27}$
$M_{J\_avg}$ [kg] =				~ $1.87 \times 10^{27}$

$R_p/R_e =$	174 mm / 186 mm $\approx$ <b>0.9355</b>
$R_{J\_avg}$ [m] =	~ <b><math>69900 \times 10^3</math></b>
$V$ [m <sup>3</sup> ] =	~ <b><math>1.43 \times 10^{24}</math></b>
$\rho_J$ [kg/m <sup>3</sup> ] =	~ <b>1310</b>

**FORMULAE**

In case of a moon orbiting a much more massive planet, the mass of the central body (planet) can be obtained from Kepler`s Third Law:

$$M_J = \frac{4\pi^2}{G} \cdot \frac{a^3}{P^2}$$

, where G – gravitational constant, a – semi-major axis of moon orbit in meters, P – period in seconds

The mean radius for a slightly oblate ellipsoid is:

$$R_{J\_avr} = \sqrt[3]{\frac{R_p}{R_e} R_e R_e^2} = R_e \times \sqrt[3]{R_p/R_e}$$